

# The Kerr–Newman Electron as a Black–White Hole: Proper Time, Projective Geometry, and Trefoil String–Quark Structure

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**Abstract.** The point electron of quantum theory, which produces Planck-scale gravity as the weakest interaction, is replaced by a super-rotating black-white hole (BWH) electron, which is based on strong Kerr–Newman (KN) Compton-scale gravity interacting with a strong electromagnetic field. Considering the BWH electron as a particle propagating along the world line  $x_\mu(\tau)$ , parameterized by the proper time  $\tau$  according to the Fock–Schwinger interpretation, we show its complete consistency with QED, as well as the following new features: (i) BWH forms an Einstein–Rosen bridge, concentrated near the region of zero proper time, with the black part of proper time  $\tau < 0$  corresponding to the positron, and the white part  $\tau > 0$  to the electron; (ii) BWH does not contradict QED by producing the anomalous magnetic moment predicted by Schwinger in 1948; (iii) BWH acquires a ring string structure formed by two Wilson loops surrounding the electron’s vacuum core; (iv) BWH creates a third ring string which turns it into a string quark with fractional electric charges, and finally Abrikosov type II superconductivity turns the BWH electron into a single trefoil.

**Keywords:** Kerr–Newman black hole; proper time; black–white hole; Wilson loops; string–quark model; anomalous magnetic moment; Ginzburg–Landau superconductivity; Abrikosov superconductivity; trefoil knot.

## 1 Introduction: from the point electron to Kerr geometry

The point electron of quantum theory was extremely successful, establishing QED as a brilliant experimentally confirmed theory, but it left aside the fundamental problem of unification quantum theory with gravity. As a result, gravity became the weakest interaction acting on the Planck scale.

The famous arguments about the weakness of gravity are indeed wrong, since they ignore the charge and giant angular momentum of the electron, which gives the point electron an infinite mass-energy and leads to an uncertainty between zero and infinity. The real scale of the Einstein–Maxwell gravity exceeds the Planck scale about  $10^{22}$  times.

The Kerr–Newman electron model, suggested by B. Carter in 1968, showed that the electron is not a point. The point quantum singularity should be replaced by the Kerr–Newman (KN) black–white hole (BWH) solution based on the Kerr gravity of Compton scale, where the strong KN gravitational field can capture the strong electromagnetic field in a single topological vortex and form the KN electron as an extended BWH vortex of Compton radius

$$a = \frac{J}{m} \simeq \frac{\hbar}{2m}, \quad (1)$$

The roots of this replacement are older than black-hole physics. Appell’s complex shift of the Coulomb potential already shows how a point singularity can be transformed into a ring singularity by complex geometry [1]. This mechanism is naturally connected with the later Kerr–Schild description of the Kerr congruence. In parallel, Heun’s theory of second-order differential equations with four branch points introduced functions that have reappeared in modern analyses of wave and Dirac equations in rotating black-hole backgrounds [2]. The old problem of a point source is therefore not merely regularized; it is transformed into a projective and analytic problem.

The projective lineage is also essential. Felix Klein’s *Vorlesungen ueber hoehere Geometrie* contains the geometry of Study lines, or bolts, as a twisting ruled surface [3]. In the author’s interpretation, this figure anticipates the geometry of the Kerr congruence: the point source is opened into a ruled, rotating, two-sheeted structure. The Kazan school of Kotelnikov and Petrov then developed the projective and algebraic classification methods that entered relativity through Pirani and Petrov’s classification of gravitational fields [4, 5]. Kerr’s 1963 solution arose from precisely this algebraically special setting [6].

The author reports that Roy Kerr himself once distinguished the two contributions by saying: “Electron ... it is yours! Mine is Kerr’s geometry.” This personal remark is not used here as evidence; it records the historical separation between Kerr’s exact geometry and the author’s electron interpretation.

The regularized KN electron model has been developed through earlier work by Carter, Israel, Debney-Kerr-Schild (further DKS), Lopez, Punsly, and the author, which in 1972 showed that Kerr gravity is really strong and is capable of keeping a photon in a ring-shaped orbit [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. Carter first emphasized the remarkable fact that the Kerr–Newman solution has the classical gyromagnetic ratio of the Dirac electron. Israel and Lopez gave source and regularization models. DKS presented consequent derivation of the Kerr–Newman solution and gave the first detailed derivation of the KN electron model. Punsly recovered the projective structure of Kerr solution, identifying it with a Klein-bottle topology formed from two projective sheets glued along a Moebius strip.

In the present Short Communication these ingredients are compressed into a final thesis: the

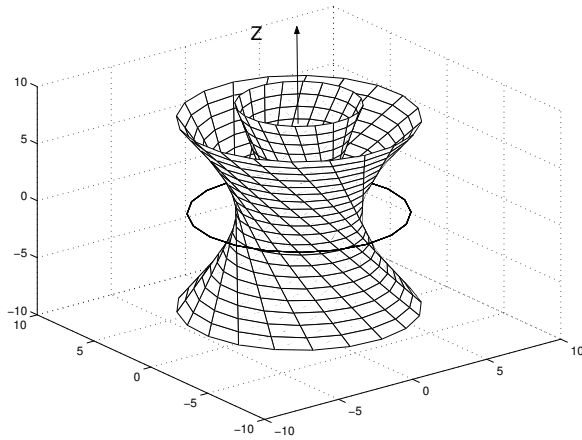


**Figure 1:** Principal null congruence of Kerr geometry reveals the inner projective structure of the point electron.

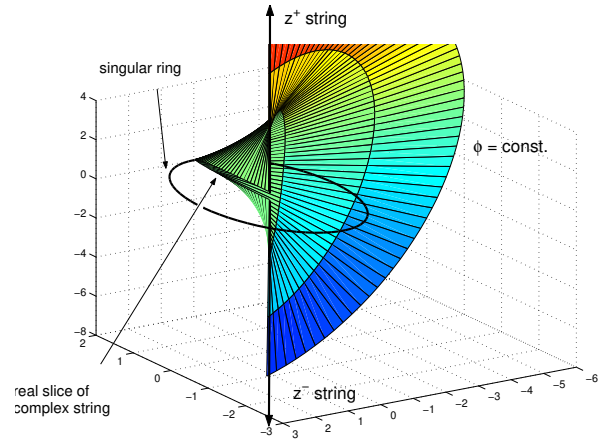


**Figure 2:** Gravitational frame-dragging pulls in the light cone near the Kerr singular ring.

free of the external interactions electron is a single horizonless gravi-electromagnetic Kerr-Newman (KN) vortex, forming a world line controlled by the proper time parameter  $\tau \in (-\infty, +\infty)$ . This world line is separated into the black part  $(-\infty, -\epsilon)$ , controlled by the past light cone  $LC_-$  and the white part  $(\epsilon, +\infty)$ , controlled by future light cone  $LC_+$ , and also in the narrow mixed region  $(-\epsilon, +\epsilon)$ , which is placed in the equatorial disk of the Kerr geometry which separates the past and future parts of the light cone at the proper time point  $\tau = 0$ .

The electron based on strong gravity does not contradict QED [18] and gives the correct anomalous magnetic moment, simultaneously forming a string-quark structure.

This note is intentionally not a long derivation. It fixes the final result in a compact form. It also records the recent proper-time insight: the electron/positron relation must be treated as a propagator of the particle parameterized by proper time, according to the interpretation of the proper time by Fock-Schwinger, Keldysh, Landau-Lifshitz and Kinnersley.

## 2 Proper time and the positron branch

The Feynman–Stueckelberg interpretation identifies the positron with an electron line whose orientation is reversed in coordinate time [19, 20]. This interpretation is powerful, but it raises an immediate conceptual question. If a particle is called a positron because it is an electron moving backward in coordinate time, what is the same particle when it continues to propagate forward after the reversal point? The answer, in the present model, is that the physical parameter of the worldline is set by the particle itself, the proper time  $\tau$  changes orientation at the moment  $\tau = 0$ , and the particle turns out to be an electron when  $\tau > 0$ , and a positron when  $\tau < 0$ .

For a timelike worldline  $x^\mu(\tau)$ , proper time is defined by

$$c^2 d\tau^2 = -g_{\mu\nu} dx^\mu dx^\nu \quad (2)$$

for signature  $(-, +, +, +)$ . In flat spacetime this reduces to

$$d\tau = dt \sqrt{1 - \frac{v^2}{c^2}}. \quad (3)$$

Fock used proper time as an invariant parameter in classical and quantum mechanics [21], and Schwinger's proper-time method later became a central tool in QED [22]. Note also earlier role of Zisman in interpretation of the positron question in 1940 [23], but the problem was considered most clear in the letter of 1969 by W. Kinnersley [24], which considered a curve parameterized by proper time and divided into past and future by a light cone. This point is central for the present paper.

The relativistic disk discussed in the Landau–Lifshitz course on classical field theory is a useful geometrical guide for this interpretation [25]. On a rigidly rotating relativistic disk the local clock rate is reduced by the factor  $\sqrt{1 - v^2/c^2}$ , and the lightlike rim is naturally associated with a limiting boundary structure. In the KN electron the ring at the Compton scale plays the analogous role: for an external observer the relativistically rotating ring string is contracted to a point-like quantum object, while in the proper-time description it remains an extended boundary of the vacuum disk.

The KN electron is therefore described as a particle propagating along a worldline  $x^\mu(\tau)$  through a two-orientation geometry created by the past and future part of the light cone. The incoming part of the worldline,  $\tau < 0$ , is interpreted as the positron branch. The outgoing part,  $\tau > 0$ , is the electron branch. The transition at  $\tau = 0$  is not a creation of a new point particle from nothing; it is the continuation of the same extended Kerr–Newman (KN) structure passing through the black–white hole (BWH).

This resolves the apparent paradox. Coordinate time may reverse its role in the observer's description, while proper time continues to order the internal propagation of the particle. In the language of the black–white hole, the black side absorbs the electromagnetic field and the white side emits it. In the language of the propagator, the incoming positron branch and the outgoing electron branch are two orientations of one proper-time object. The Fock–Schwinger parameter is thus not a technical convenience but the natural time variable of the KN electron.

### 3 The Kerr–Newman BWH as an exact strong gravity solution of the Einstein–Maxwell field equations

The Kerr–Newman metric can be written in the Kerr–Schild form (DKS) as follows

$$g_{\mu\nu}^\pm = \eta_{\mu\nu} + 2H k_\mu^\pm k_\nu^\pm, \quad H = \frac{mr - e^2/2}{r^2 + a^2 \cos^2 \theta}, \quad (4)$$

where  $k_\mu^\pm$  are the two generators of the future and past null cones. The electromagnetic potential may be written as

$$A_\mu^\pm dx^\mu = \frac{er}{r^2 + a^2 \cos^2 \theta} k_\mu^\pm dx^\mu. \quad (5)$$

For electron parameters the geometry is superextremal,  $a^2 + e^2 \gg m^2$ , so it has no event horizon in the standard black-hole sense. The phrase black–white hole is used here for the paired ingoing/outgoing Kerr–Schild configuration and for its absorption–emission interpretation, not for the presence of an ordinary event horizon.

Lopez's regularization, [10], selects the boundary of the vacuum core at the point where  $H = 0$ ,

giving

$$r = r_e = \frac{e^2}{2m}. \quad (6)$$

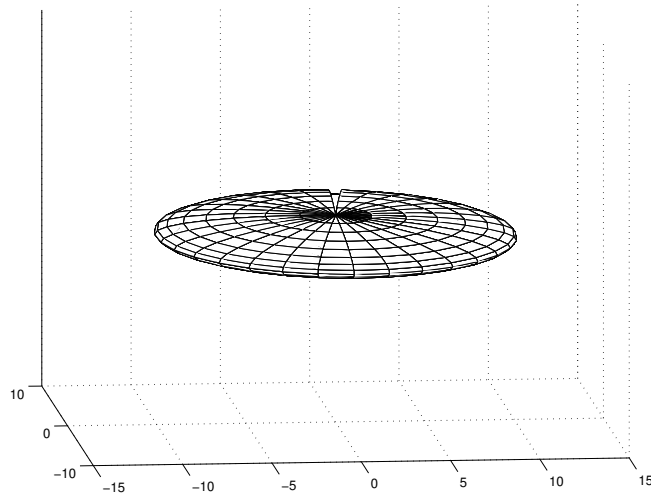


Figure 3: Disk-like vacuum core of the regularized KN solution.

The Lopez boundary actually turns the KN solution into an exact solution to the Einstein-Maxwell field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi T_{\mu\nu}. \quad (7)$$

The exact KN solution is based on the Ricci-flat geometry which corresponds to the zero scalar curvature,  $R = 0$ , and as a result, the trace of the energy-momentum tensor should also vanish,

$$T = T_{00}g^{00} + T_{11}g^{11} + T_{22}g^{22} + T_{33}g^{33} = 0. \quad (8)$$

This equation is well known as the gravitational anomaly, which is the main obstacle to the quantization of gravity! At the same time, this turns out to be extremely important for the main thesis of this paper, since it gives the KN solution a new status, turning the KN electron into a string quark created by strong gravity.

The KN electron has no mechanical mass, and similar to the spherical point-like electron, the full mass-energy of the electron is created by *the time component of the retarded potential of the electromagnetic field* [26, 25],

$$m = U = \frac{e}{2} \int_{r_e}^{\infty} A_t(r) dr. \quad (9)$$

Unlike a point quantum electron, the charge of a KN electron is distributed along the boundary of a vacuum core, similar to the model of a relativistically rotating disk given by Born and the interpretation of proper time given in the Course on Relative Field Theory [25]. As a result, the full mass-energy of the KN electron is controlled by the charged ring string placed at the radial distance  $r_e$  (6).

Thus, the third ring string, created by the time component of the retarded potential  $A_t(r_e)$ , controls the incoming and outgoing energy balance in BWH. The other two ring strings are two

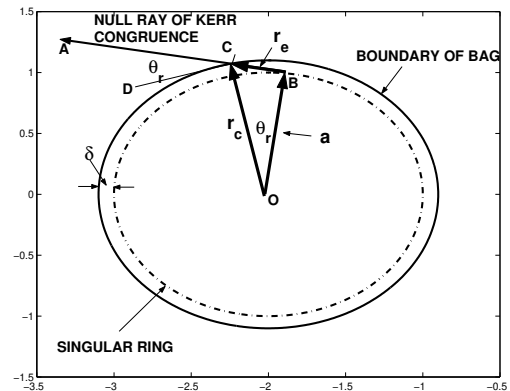


Figure 4: Vector potential of KN solution is concentrated in equatorial plane, on the border of vacuum core.

Wilson loops formed by the spatial components of the potentials.

The vacuum core of the KN electron is not empty in the physical sense, but forms a disk-shaped vacuum region formed by a Ginzburg–Landau superconducting phase [27]. This phase expels the electromagnetic field from the interior of the vacuum core to the boundary (6) where the strong KN field concentrates the potential and current on two ring like  $\delta$ -distributions, forming the incoming and outgoing Wilson loops  $C_{\mp}$  as two ring strings  $S_{\mp}$  dressed by gravity [16, 17].

Abrikosov type-II superconductivity then appears at the border of the vacuum disk in equatorial plane, where vortex-string flux is localized [28]. In this way the regularized KN electron takes the form of a thin superconducting vacuum bag [29] covering the former singular ring and separated from the external gravity and electromagnetic field by a relativistically rotating domain wall. In the equatorial plane of the bag boundaries  $B_{\mp}$ , the incoming and outgoing potentials take opposite maximum absolute values  $A_{\mu}^{max\pm}$ , which are dragged by the strong vortex gravity around the border of vacuum core, forming two oppositely charged and oppositely oriented Wilson loops  $C_{-}$  and  $C_{+}$

$$eA^{max\pm}|_{C_{\pm}} = \pm 2ma \, d\phi. \quad (10)$$

Therefore

$$\oint_{C_{\pm}} eA^{max\pm} = \pm 4\pi J, \quad J = ma. \quad (11)$$

Two boundary fields of opposite charges transported around the incoming and outgoing loops acquire two opposite phases

$$\gamma_{\pm}(e) = \frac{e}{\hbar} \oint_{C_{\pm}} A^{max\pm} = \pm 4\pi \frac{J}{\hbar}. \quad (12)$$

Single-valuedness requires  $\gamma_{\pm}(e) = 2\pi n$ , and gives

$$J = \frac{n\hbar}{2}. \quad (13)$$

The values  $n = 1, 2, 3$  indicate that a similar structure should arise for leptons  $\mu$  and  $\tau$ . Thus the half-integer spin of the electron appears as a boundary holonomy condition of the KN vacuum core.

The two holonomies have opposite orientation: at  $\tau < 0$  the particle goes through the black branch of the light cone which drags the incoming electromagnetic potential into the rotating core, and at  $\tau > 0$ , the white branch of the light cone emits the outgoing potential. The two loops are not external decorations of the electron, approaching the boundary of vacuum core, they form extreme strong concentration of the incoming potential  $A_{\mu}^{max-}$  on the equatorial plane forming the incoming Wilson loop as a string carrying negative mass contribution

$$m_{-} = \frac{e^2}{2r^{-}}. \quad (14)$$

On the white side of the light cone, the potential is controlled by generators of the outgoing null field, which is carried by the outgoing potential  $A_{\mu}^{max+}$ , forming an outgoing Wilson loop in the equatorial plane in the form of a string with positive mass contribution

$$m_{+} = \frac{e^2}{2r^{+}} \quad (15)$$

The superconducting vacuum bag is bounded by two Wilson loops, which are considered to be its

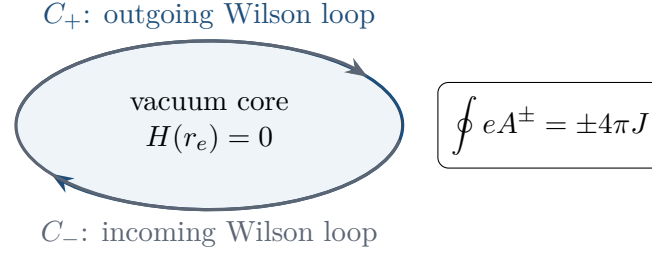


**Figure 5:** Schematic of the two oriented Wilson loops along the boundary of the vacuum core. Their opposite phases encode the black–white pair of the two Kerr–Schild branches.

string structure.

Thus, the vacuum core is formed as a magnetically coupled pair of the positive mass contribution  $m_+$ , created by the Wilson loop  $C_+$  with positive potential  $A_\mu^{max+}$ , and the negative mass contribution  $m_-$ , created by the oppositely charged Wilson loop  $C_-$  with potential  $A_\mu^{max-}$ , forming a magnetically coupled pair of monopole and antimonopole [16, 30].

#### 4 The anomalous magnetic moment

The first remarkable success of the KN electron is already classical. The asymptotic magnetic moment of the KN solution is

$$\mu = ea = \frac{eJ}{m}, \quad (16)$$

which corresponds to the Dirac gyromagnetic ratio  $g = 2$  [7]. This fact was Carter’s original reason for taking the KN electron seriously as a geometric model of the charged spinning lepton.

The stronger claim of the present black–white completion is that the extended source also supplies the anomalous magnetic moment predicted by Schwinger in 1948,

$$a_e \equiv \frac{g-2}{2} = \frac{\alpha_{\text{fs}}}{2\pi} + O(\alpha_{\text{fs}}^2), \quad (17)$$

where  $\alpha_{\text{fs}}$  is the fine-structure constant [31]. In ordinary QED this term is obtained from the one-loop vertex correction. In the KN model, the same number appears as a direct consequence of the trace anomaly condition (8), which establishes that the KN solution is in fact an exact solution to the Einstein–Maxwell system of equations. The new radial coordinate  $r_e$  regularises the Kerr singular ring, setting the boundary of vacuum core beyond the Compton radius (1), forming a regular relativistically rotating vacuum bag that covers the Kerr singular ring.

The important conceptual distinction is the following. The exact KN exterior fixes the Dirac value  $g = 2$ . The black–white string completion is proposed to generate the finite anomalous contribution. The anomaly is therefore not introduced as an arbitrary parameter. It is tied to the same Compton-scale geometry that produces the ring singularity, the Lopez boundary, and the Wilson-loop phases, see [18].

Thus the anomalous moment becomes a decisive test of the model, the superconducting KN core and two ring strings produce the Schwinger coefficient, the BWH electron supplies a geometric origin for a central QED observable.

## 5 From two Wilson loops to the trefoil string–quark

The two Wilson loops  $C_-$  and  $C_+$  are the first string structure of the KN electron. We also denote the corresponding oriented string components by  $S_-$  and  $S_+$ . The black side of the BWH drags the incoming electromagnetic potential into the rotating core, forming the incoming ring string  $S_-$ . The white side creates the outgoing Wilson loop, pushed out of the core as the second ring string  $S_+$ . These two strings correspond to the two time orientations and to the two principal Kerr congruences.

However, the black–white electron is not complete with only two loops. The strings  $S_-$  and  $S_+$  have opposite orientations. Taken by themselves, they leave the boundary phase as an open two-string system: in the author’s terminology, the opposite orientations break the gauge invariance of the two-string configuration. At the transition between absorption and emission, the two oppositely directed streams meet. In the author’s description this event is an instanton-like flash of vacuum projected to future infinity. The particle must continue from  $\tau < 0$  to  $\tau > 0$  with the same mass-energy and with a consistent outgoing branch.

A third ring string is therefore required. Its first role is dynamical: it compensates the reversed path and completes the propagation into the future. Its second role is gauge-theoretic: it restores gauge invariance by closing the total Wilson circuit of the black–white boundary system. Thus the BWH completion is not an optional decoration of the two Wilson loops; it is the closure condition that turns the two oppositely oriented strings into one gauge-consistent object.

This third string, as the first two, is created by the gravitational anomaly equation (8), which gives as before the Lopez boundary (6), but in fact, the situation changes dramatically, since the third string forms a fractionally charged system of three strings  $S_0, S_+, S_-$ , which transform the KN electron into a string quark with charges of equal modulus  $e^0 = e^- = -e^+ = -\frac{1}{3}|e|$ .

The mass of the positron is positive, just like the mass of the electron. The negative contribution to the electron’s mass-energy,  $m_-$ , can be viewed as dark energy connecting the incoming and outgoing strings  $S_-$  and  $S_+$ , leading to the formation of the electron’s vacuum core upon the annihilation of masses  $m_-$  and  $m_+$ . The mass defect is determined by the string  $S_0$ , which restores the gauge invariance of the bosonic sector, provided the balance condition  $m_0 - m_+ = 0$  holds.

In this interpretation the fractional electric charges are not independent point-particle charges. They are effective charges of the three-string electromagnetic configuration bound to the black–white Kerr core.

The superconducting mechanism has two layers. The Ginzburg–Landau superconducting phase forms the disk-like vacuum core and expels the electromagnetic field from the interior on the border (6) [27]. Abrikosov type-II superconductivity appears on the border of this vacuum disk and confines magnetic flux into vortex-string structures [28]. The two Wilson-loop strings and the third BWH string are then joined into a single superconducting gluon. Topologically, the smooth closure of the gluon gives a trefoil knot. We emphasize that this topological string quark structure of the electron, extended to  $\pm\infty$ , assumes that the electron is a free particle, controlled by proper time and isolated from external fields and interactions. In the opposite case, for example, when considering an electron in an atom, the points  $+\infty$  and  $-\infty$  can be topologically joined within the atom and form a vast superconducting vortex region of the atom size where the gluon string of type II superconductivity will form its own path.

The physical description of the high-energy BWH electron depends on the level of high energy: whether it is a single atom, radar system, laser, or atomic reactor, the quantum description will

vary depending on the level of energy, transitioning into a thermal form of a semiclassical quantum theory, although formally it can still be described as a single Fock propagator from  $t = -\infty$  to  $t = +\infty$  or as a "Keldysh rotation" forming a symmetric  $phy^+$  and an antisymmetric  $phy^-$  form from the Fock propagators, as described in the Schwinger–Keldysh formalism (see, for example, [32]).

## 6 What this Short Communication fixes

This Short Communication fixes four theses.

First, the electron is not treated as a point source whose gravity becomes relevant only at the Planck scale. It is treated as a Kerr–Newman extended source in which strong gravity acts at the Compton scale.

Second, the electron/positron relation is organized by Fock proper time. The incoming branch  $\tau < 0$  is the positron branch, and the outgoing branch  $\tau > 0$  is the electron branch. This gives a modification of the Feynman–Stückelberg idea within the proper-time framework of black and white KN geometry.

Third, the KN vacuum core carries boundary Wilson loops. Their holonomies give the half-integer angular-momentum condition when the boundary charge is the electron charge. The spin is therefore attached to a gauge phase of the Kerr–Newman boundary.

Fourth, the modified black–white geometry controlled by the balance equation (8) adds a third string. The Ginzburg–Landau superconducting phase forms the disk-like vacuum core, while Abrikosov type-II superconductivity localizes the boundary vortex-string structure. The third string closes the two oppositely oriented Wilson loops, restoring the gauge-invariant boundary circuit. It turns the two-loop KN electron into a trefoil string–quark with fractional charge structure.

These statements do not exhaust the required mathematics. They identify the final structure to be derived in full.

The present paper is the concise theoretical synthesis that fixes the path to the goal.

## 7 Conclusion

The contradiction between quantum theory and Einstein gravity is overcome, in this model, by abandoning the point electron and replacing it with an extended Kerr–Newman black–white particle. The electron is a horizonless, Compton-scale object whose geometry is controlled by the two principal Kerr congruences, whose core is regularized by the Lopez boundary, and whose electromagnetic phase is carried by Wilson loops.

The decisive conceptual step is the use of proper time. The incoming part of the worldline,  $\tau < 0$ , is identified with the positron branch, while the outgoing part,  $\tau > 0$ , is identified with the electron branch. The black–white geometry therefore supplies a single proper-time object behind the electron/positron pair.

The two Wilson loops surrounding the vacuum core give the ring-string structure of the KN electron and the half-integer angular-momentum condition. The Ginzburg–Landau superconducting phase forms the disk-like vacuum core. The black–white modification then adds a third ring string at the boundary, where Abrikosov type-II superconductivity confines the vortex-string flux. This third string closes the oppositely oriented pair  $S_-$  and  $S_+$  and restores the gauge-invariant superconducting circuit. The three strings form a single trefoil. The resulting string–quark structure

of the KN electron, based on QED-compatible strong BWH Compton-scale gravity, can replace not only the weak QED Planck-scale gravity, but also the strong gravity of the Standard Model.

## Declarations and editorial provenance

**Relation to Prior Work.** This Short Communication is a compact authorial synthesis emphasizing the proper-time, superconducting-core, gauge-closure, and trefoil/string-quark formulation. It builds on and acknowledges the author’s earlier Kerr–Newman electron publications [14, 15, 16, 17] and the broader black–white/string–quark construction presented in conference form [33]. The contribution of the present Short Communication is the compact integration of these elements into the proper-time, superconducting-core, gauge-closure, and trefoil string–quark formulation stated by the author.

**Data and code availability.** No new dataset or software is used. The work is analytic and conceptual.

**Competing interests.** The author declares no competing interests.

**Transparency statement.** The scientific content and conclusions remain the author’s responsibility. Gauge Freedom Journal editorial support, including AI-assisted language editing, consistency review, and L<sup>A</sup>T<sub>E</sub>X production, was used in preparing this article.

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